DL-Lite and role inclusions

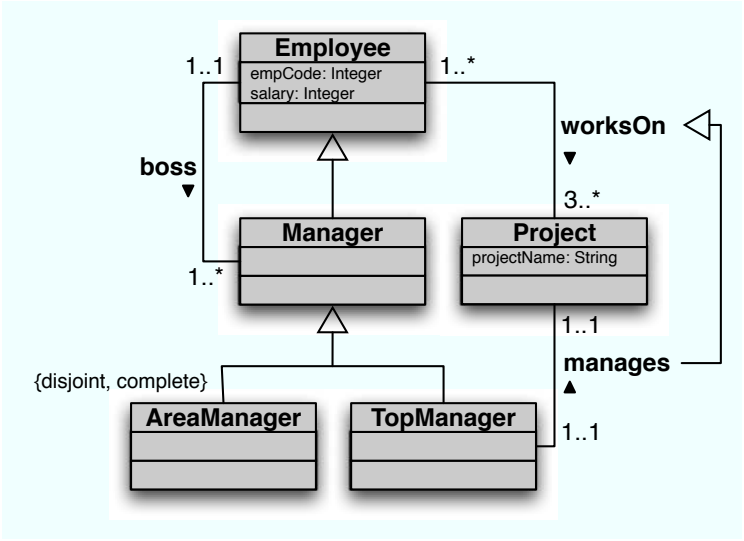
Roman Kontchakov and **Michael Zakharyaschev**

SCSIS, Birkbeck College, London

http://www.dcs.bbk.ac.uk/~roman

DL-Lite: Description Logic for Modelling

A fragment of
a UML class diagram:



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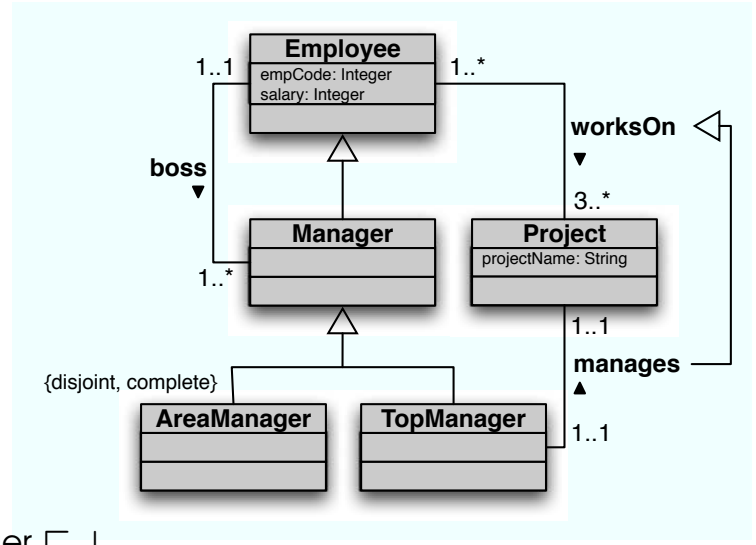

Translating into DL:

$\mathsf{TopManager} \sqsubseteq \mathsf{Manager}$

$\mathsf{AreaManager} \sqcap \mathsf{TopManager} \sqsubseteq \bot$

$\mathsf{Manager} \sqsubseteq \mathsf{AreaManager} \sqcup \mathsf{TopManager}$

$\exists \mathsf{worksOn}.\top \sqsubseteq \mathsf{Employee}$ (role typing)

$\exists \mathsf{worksOn}^{-}.\top \sqsubseteq \mathsf{Project}$

$\mathsf{Project} \sqsubseteq \geq 3\, \mathsf{worksOn}^{-}.\top$ (cardinality constraints)

. . .

$\mathsf{manages} \sqsubseteq \mathsf{worksOn}$ (role inclusions)

DL-Lite in OWL 2

OWL 2 QL, one of the three profiles of OWL 2, is based on a dialect of *DL-Lite*:

> "OWL 2 QL is aimed at applications that use very large volumes of instance data, and where query answering is the most important reasoning task. In OWL 2 QL, conjunctive query answering can be implemented using conventional relational database systems, and can directly access data stored in such systems. Using this technique, sound and complete query answering can be performed in LogSpace with respect to the size of the data (assertions). ...there are polynomial time algorithms for consistency, subsumption, and classification reasoning."

NB: data complexity higher than LogSpace would mean that relational database systems could not be directly used for query answering (e.g., DataLog is data-complete for polynomial time)

OWL 2 QL

Subclass-Expression: a class or
existential quantification (**ObjectSomeValuesFrom**) with the class limited to **owl:Thing**

Superclass-Expression: a class,
existential quantification to a class (**ObjectSomeValuesFrom**),
negation (**ObjectComplementOf**) or intersection (**ObjectIntersectionOf**)

OWL 2 QL has the following features:

- subclass axioms (**SubClassOf**)
- class expression equivalence (**EquivalentClasses**)
- class expression disjointness (**DisjointClasses**)
- inverse object properties (**InverseObjectProperties**)
- property inclusion (**SubObjectPropertyOf** not involving property chains and **SubDataPropertyOf**)
- property equivalence (**EquivalentObjectProperties** and **EquivalentDataProperties**)
- property domain/range (**ObjectPropertyDomain/Range**, **DataPropertyDomain/Range**)
- disjoint properties (**DisjointObjectProperties** and **DisjointDataProperties**)
- symmetric properties (**SymmetricObjectProperty**)
- assertions other than the equality assertions (**DifferentIndividuals**, **ClassAssertion**, **ObjectPropertyAssertion**, and **DataPropertyAssertion**)

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WHY THESE?
can it be extended?

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The *DL-Lite* family with role inclusions

1. *DL-Lite*$_{bool}^{\mathcal{R}}$

$$
\begin{array}{lcl}
R & ::= & P \mid P^- \\
B & ::= & \bot \mid A \mid \exists R \\
C & ::= & B \mid \neg C \mid C_1 \sqcap C_2
\end{array}
$$

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combined complexity sat.: **NP**
data comp. inst. check.: **LogSpace**
data comp. query ans.: **coNP**

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combined complexity: **P**
data comp. instance: **LogSpace**
data comp. query: **LogSpace**

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TBox axioms $B_1 \sqsubseteq B_2$ $B_1 \sqsubseteq \neg B_2$ $\neg B_1 \sqsubseteq B_2$

comb. comp.: **NLogSpace**
d.c. instance: **LogSpace**
d.c. query: **coNP**

4. *DL-Lite*$_{core}^{\mathcal{R}}$ = *DL-Lite*$_{horn}^{\mathcal{R}}$ ∩ *DL-Lite*$_{krom}^{\mathcal{R}}$

comb. comp.: **NLogSpace**
d.c. instance: **LogSpace**
d.c. query: **LogSpace**

NB: **LogSpace** upper bounds by embedding in the 1-variable fragments of FOL

NB: results **do not** depend on the Unique Name Assumption (**UNA**)

Extensions of *DL-Lite*

- $\mathit{DL\text{-}Lite}_{core}^{\mathcal{R},\mathcal{N}}$: number restrictions + role inclusions $\rightsquigarrow$
  combined complexity EXPTIME; data complexity coNP

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- $\mathit{DL\text{-}Lite}_{core}^{\mathcal{R}}$ + transitive roles $\rightsquigarrow$ data complexity NLOGSPACE

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- *DL-Lite*$_{core}^{\mathcal{R}}$ **+** transitive roles $\rightsquigarrow$ data complexity NLOGSPACE

- The following **doesn't** change complexity of *DL-Lite*$_{\alpha}^{\mathcal{R}}$:

  $\surd$ symmetric roles ($\boldsymbol{P} \sqsubseteq \boldsymbol{P}^{-}$), asymmetric roles
  $\surd$ reflexive roles, irreflexive roles
  $\surd$ disjoint roles
  $\surd$ equality assertions ($\boldsymbol{a} \approx \boldsymbol{b}$)

Conclusions

- obtained first proofs that the *DL-Lite* logics with role inclusions and functionality/number restrictions are indeed not first-order reducible (provided that P $\neq$ LogSpace)

- showed that conjunction can be used on the left-hand side of concept inclusions without affecting data complexity ($\mathcal{DL}$-*Lite*$_{horn}^{\mathcal{R}}$ vs. $\mathcal{DL}$-*Lite*$_{core}^{\mathcal{R}}$)

- identified a number of extensions that could be included in OWL 2 QL without damaging good computational properties

- more on *DL-Lite*:
  A. Artale, D. Calvanese, R. Kontchakov and M. Zakharyaschev.
  DL-Lite in the Light of First-Order Logic.
  In Proc. of the 22nd AAAI Conference on Artificial Intelligence, pp. 361–366.
  AAAI Press, 2007.